OT TECHNICAL MEMORANDUM 77-235

A CALIBRATED TEST RANGE FOR EVALUATION OF REFRACTION CORRECTION METHODS

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U.S. DEPARTMENT OF COMMERCE / Office of Telecommunications

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M.C. Thompson, Jr.

L.E. Wood

K.C. Allen

U.S. DEPARTMENT OF COMMERCE
Juanita M. Kreps, Secretary

Jordan J. Baruch, Assistant Secretary
for Science and Technology

OFFICE OF TELECOMMUNICATIONS
John M. Richardson, Director

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UNITED STATES DEPARTMENT OF COMMERCE
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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

A CALIBRATED TEST RANGE FOR EVALUATION OF REFRACTION CORRECTION METHODS

M. C. Thompson, L. E. Wood, and K. C. Allen*

A 64 km path between the islands of Hawaii and Maui was measured using a Geodimeter with refractive index correction based on airborne measurements of temperature along the line. The standard deviation of the slant range is estimated as about 0.9 ppm (0.06 m).

1. INTRODUCTION

To approach the potential instrumental accuracy of microwave ranging systems, the measurements must be adjusted for the signal velocity over the path being measured. This velocity, v, is commonly considered in terms of the refractive index, n, or the refractivity, N, of the medium through which the signal is propagated. In the earth's atmosphere for systems using radio frequencies up to 40 GHz, these quantities are essentially determined by the state and composition of the air as expressed in the following:

$$n \equiv \frac{c}{v} \tag{1}$$

$$N \equiv (n - 1) \times 10^6 \tag{2}$$

$$N = \frac{77.6P}{T} + \frac{3.73 \times 10^5 e}{T^2} \tag{3}$$

where P and e are total pressure and water vapor pressure, respectively, in millibars, and T is temperature in degrees Kelvin.

In the atmosphere, all three of these quantities vary in space and time. Various techniques have been used to estimate the value of N appropriate for the correction in individual cases. These methods vary in the kinds of input data they

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*The authors are with the U. S. Department of Commerce, Office of Telecommunications, Institute for Telecommunication Sciences, Boulder, Colorado 80302.

require, the approximations involved, and the methods used in calculation of the correction factors and, consequently, in the degree to which they succeed in approximating the correct value of the required correction factor.

The test range described in the following was established to provide a means for evaluating the accuracy of various correction methods. It has been comparatively easy to determine the precision obtainable from these methods. However, only a few lines in the world have been measured with the accuracy required to evaluate the residual systematic errors within 1 to 2 parts per million. Furthermore, for most tracking and many geodetic applications, the evaluations should be made over a path from 10 to 100 km long, and along which the refractive index undergoes appreciable variations in space and time.

2. PATH GEOMETRY

Such a path has been established between Mt. Haleakala, Maui, Hawaii, and Upolu Point, near the northwestern extremity of the Island of Hawaii.

The 64-km path extends from about 3000 m to 30 m. mean sea level elevation with an elevation angle of about 2.5° as shown in Figures 1 and 2, and simulates aircraft-, missile-, or satellite-to-ground geometry.

3. MEASUREMENTS

The calibration was made using an AGA Model 8 geodimeter modified by the Coast and Geodetic Survey (currently National Ocean Survey) to use a 5 mW He-Ne laser in place of the normal 1 mW unit. The reflector consisted of 51 two-inch corner cubes at the 3000 m terminal (C&GS benchmark "KOLEKOLE 13"). Corrections for these measurements were obtained from temperature measurements taken by an instrumented aircraft which was flown along the line. Since a cloud-free path was required for the optical measurement, it was usually feasible to fly nearly directly between the terminals. Figure shows samples of the temperature recordings.

A total of 16 sets of data were taken during eight days of the period from August 17 to August 26, 1971, and these were fairly evenly distributed around the clock. In general, the visibility along the path was either "zero" or good; i.e., in the absence of clouds, the light levels were adequate to give stable readings. This is indicated more quantitatively by the relatively small differences between readings; e.g., 30 mm or 0.5 ppm in each set.

4. DATA REDUCTION AND ERROR ANALYSIS

The geodimeter data were reduced in accordance with the standard procedures to obtain the uncorrected length measurement, D, for each flight. A refractivity correction, $\overline{N}$, was determined for each flight by the procedure described in the following and used to obtain an adjusted length, D_R (see Table I). The mean of these values, $\overline{D}_R$, was then adjusted for the other instrumental corrections and path curvature to obtain the estimate of the slant path length, D_C .

Table II summarizes the correction terms as well as the estimate of their standard deviations.

4.1 Instrument Corrections

4.1.1. Instrument constant

The geodimeter (No. 80070) had an instrument constant of 20.3 cm.

4.1.2. Reflector constant

The retrodirective prisms introduce an additional path length of 3.0 cm with an assumed standard deviation of 0.3 cm.

4.1.3. Modulation frequency

The instrument calibration is based on a velocity of light in a vacuum of 299,792.5 km/s, atmospheric refractivity of 308.6 N, and a modulation frequency (F1) of 29.970 MHz (wavelength of 10 m). During the time observations were being made, 23 measurements of F1 were made with a standard deviation of 0.10 ppm. Since F1 was measured for only some of the observations sets, the sets were not individually corrected. Instead, the mean of the F1 measurements was used to correct for modulation frequency. The resulting correction was 0.68 ppm and, including the errors in the frequency counter, the standard deviation assumed for this correction is 0.5 ppm.

4.1.4. Instrument calibration

The geodimeter operating manual (AGA publication 571/2501) gives the following uncertainties in the calibration and use of the instrument:

Geodimeter constant: ± 2 mm.

Eccentricities of geodimeter and reflectors: ± 1 mm.

Phase determination: ± 3 mm.

In estimating the overall measurement precision, each of these numbers is taken as standard deviation, σ.

4.2 Average Path Refractivity

4.2.1. Temperature

For each set of geodimeter observations, there were two temperature profiles, one for the ascent of the aircraft along the path and one for the descent along the path. Figure 4 is a sample of such a pair of temperature profiles, together with the calculated refractivity profiles.

Several methods were used to correct for aerodynamic heating of the temperature probe due to the airspeed of the aircraft. (NBS Report, 1969, and Kelly and Brean 1967). The resultting correction was 1.6°C with an estimated σ of 0.4°C. The σ for the probe calibration was assumed to be 0.1°C (0.1 ppm).

4.2.2. Pressure

Pressure profiles were calculated approximating the NACA Standard Atmosphere (Smithionan Institute, 1965) by

$$P(\text{in.Hg}) = 29.92\,[1 - 6.879 \times 10^{-6} Z\ (\text{ft})]^{5.2553} \quad .$$

For each flight, the pressure at ground level (100 ft or 30 m) was recorded. The calculated (standard atmosphere) pressure for Z = 100 ft (30 m) was subtracted from the ground measurement and the difference was added to the standard pressure at the remaining height intervals. The standard deviation of the results is assumed as 1 mb.

4.2.3. Water vapor

Since its contribution to refractivity at this wavelength (6328Å) is small, the water vapor was not measured along the path. A value of 50% average relative humidity along the path was assumed, giving a correction of 0.03 m with an assumed standard deviation of 1.3 cm.

4.2.4. Calculation of refractivity

The temperature and pressure profiles were used to calculate N-profiles from the formula (Eden, 1966; Wood and Thompson, 1968)

$$N = 3729 \frac{P(\text{in.Hg})}{T(°K)} .$$

4.2.5. Altitude adjustment

The altimeter data were adjusted for temperature and relative humidity in the following way.

Humidity profiles were available in 13 cases during the measurements. An average profile was derived from these and used to calculate a virtual temperature profile, $T_i(Z)$, for one of the flights. From this, the average virtual temperature, T_{mv}, from 100 ft. to the n-th altimeter reading ($n > 3$) was obtained as

$$T_{mv} = \frac{0.4T_1 + 0.8T_2 + T_3 + T_4 + \cdots + 0.5T_n}{n - 1.2} .$$

The corrected altitude, A, was calculated from

$$A = E + T_{mv} \left(\frac{I-E}{T_{mx}} \right)$$

where the values of the term $\frac{I-E}{T_{mx}}$ were obtained by interpolation from the NACA standard atmosphere.

The difference between mean temperature and mean virtual temperature at 10,000 ft. was 1.54°C, which was added to the mean temperature of each profile to approximate the mean virtual temperature. This permitted calculation of the maximum height of each profile taken during the geodimeter observations.

This maximum height was divided into 20 equal intervals for the ray tracing calculations. The assumed σ of these calculated maximum heights is 80 ft. This would result in a corresponding σ in average N (from ray tracing) of 0.3 N.

4.2.6. Ray tracing

The average value of refractivity along the path, N_p, was calculated from the electrical length, L, and the geometrical length, S, using the relation

$$N_p \equiv \frac{L-S}{S} \times 10^6 .$$

Both L and S were determined by ray tracing in the following manner. The terminal heights were taken as 30 and 3052 m. (from surveys of the airport and the "KOLEKOLE 13" benchmark, respectively).

For refractivity profile 1, the ground distance, X_T, and the take-off angle, α_o, were estimated and used to calculate the electrical length, L. The values of X_T and α_o were then varied until the calculated L was within .006 m of the mean of the geodimeter data obtained during the run, and the final elevation was within 0.06 m of the geodetic elevation (3052 m). This gave X_T = 64,241.9 m.

For profile -1 (the first descent), the process was repeated until the lengths were within 0.024 m and the elevation within 0.20 m for which X_T = 64,241.9 m also.

For the remaining profiles, X_T was taken as the value obtained from profiles 1 and -1, and α_o was adjusted until the terminal elevation of the ray was within 2 m of the geodetic value.

The sensitivities of N_p calculated this was are about 0.01 N/m for Z_T (less for X_T) and about 0.6 N/mrad for α_o.

The assumed standard deviation of N_p from the ray tracing is 0.2 N.

Errors are also introduced by the differences in time and space between the flight path of the aircraft and the actual beam path of the geodimeter. Assuming that the profile measured by the aircraft varied from the actual path profile randomly at each point with a standard deviation of 2N (considered to be a conservative estimate), the contribution to the standard deviation of $\overline{D}_R$ is 0.08 ppm, or 5 mm.

4.3 Correction for Refractivity

The correction of the geodimeter observations by the calculated average path refractivity was done in the following way.

For each observation set, there are two path averages (ascent and descent). The 2 path averages were averaged together to give one correction term, $\overline{N}$, for each observation set. Since the geodimeter has a built in refractivity term of 308.6 N, the following formula was used to correct the mean, D, of each observation set:

$$D_R = D \frac{1.0003086}{1 + \overline{N} \times 10^{-6}} .$$

Table I is a summary of the refractive index corrections. The values of D and $\overline{N}$ are plotted vs time in Figure 8 and cross-plotted in Figure 9 to illustrate their correlation.

4.4 Geometric Adjustments

4.4.1. Instrument tilt

The tilt scale reading was + 0.8 cm, and the standard deviation in path length resulting from uncertainties in this reading is assumed as 0.1 cm.

4.4.2. Reflector tilt

The reflector at "KOLEKOLE 13" was rotated about the benchmark to an inclination (from the vertical) of 8.5°. The correction for this inclination is 4.5 cm. with an assumed standard deviation of 0.5 cm.

4.4.3. Path curvature

To reduce the arc length to the slant range, the radius of curvature ρ was taken as the reciprocal of the index gradient between the two terminals;i.e., $\rho = - \frac{(Z_{10,000} - Z_{100})}{(N_{10,000} - N_{100})}$.

The correction term C is given as

$$C = \frac{\rho \sin \phi}{\cos \frac{\phi}{2}} \quad \text{where} \quad \phi = \frac{\text{arc length}}{\rho} .$$

Values of C calculated from 6 profiles (1 through -3) gave a mean value of 0.8 cm. with a standard deviation of 0.1 cm.

4.5 Conclusion

Table II summarizes the corrections involved in arriving at the estimated slant distance and the observed or assumed standard deviations of the corrections. They are divided into two categories, those associated with the measurement of the electrical length and those involved in the estimate of the atmospheric retardation. The resulting estimate of the slant range is 64,313.338 m with standard deviation of 0.057 m or 0.89 ppm.

5. ACKNOWLEDGEMENTS

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TABLE I

SUMMARY OF UNCORRECTED MEASURED LENGTH D, REFRACTIVITY CORRECTIONS, $\bar{N}$, AND LENGTH CORRECTED FOR REFRACTIVITY, D_R .

FLT. NO.	D	$\bar{N}$	D_R
1	64308.738	240.11	64313.142
3	64308.836	241.29	64313.164
4	64308.866	242.40	64313.123
5	64308.851	241.51	64313.164
6	64308.804	240.82	64313.162
7	64308.819	241.24	64313.150
8	64308.829	241.57	64313.139
9	64308.801	241.20	64313.134
10	64308.770	240.41	64313.154
11	64308.828	241.00	64313.174
12	64308.788	240.89	64313.141
13	64308.790	240.93	64313.141
14	64308.770	240.60	64313.142
15	64308.792	240.63	64313.162
16	64308.757	240.20	64313.154
MEAN:	$\bar{D}$ = 64308.805	$\bar{N}_{mean}$ = 240.909	$\bar{D}_R$ = 64313.150
SD:	0.035 m (0.54 ppm)	0.621 (N-units)	0.014 m (0.22 ppm)
		σ:	0.004 m (0.06 ppm)

$\rho_{D,\bar{N}} = 0.931$

TABLE II

SUMMARY OF CORRECTION TERMS

GEODIMETER MEASUREMENT		CORRECTION	STANDARD DEVIATION
Geodimeter Constant		0.203 m	0.002 m
Reflector Constant		-0.030	0.003
Modulation Frequency Corr.		-0.044	0.038
Reflector Tilt Correction		0.045	0.005
Eccentricities of Geodimeter		-	0.001
Phase Determination		-	0.003
Tilt Scale Reading		-0.008	0.001
Subtotal		0.166	0.039
REFRACTIVITY MEASUREMENT			
Aerodynamic Heating Corr.		-	0.024
Temperature Probe Accuracy		-	0.006
Pressure Accuracy		-	0.019
Water Vapor Correction		0.030	0.013
Altitude Corrections		-	0.019
Ray Tracing		-	0.013
Path Curvature Correction		-0.008	0.001
Aircraft Path		-	0.007
Determination of the Mean $\overline{D}_R$		-	0.004
Subtotal		0.022	0.042
Total		0.188	0.057
Mean length after refractivity adjustment	$\overline{D}_R$	64313.150	
Mean slant path length	D_C	64313.338 m	0.057 m

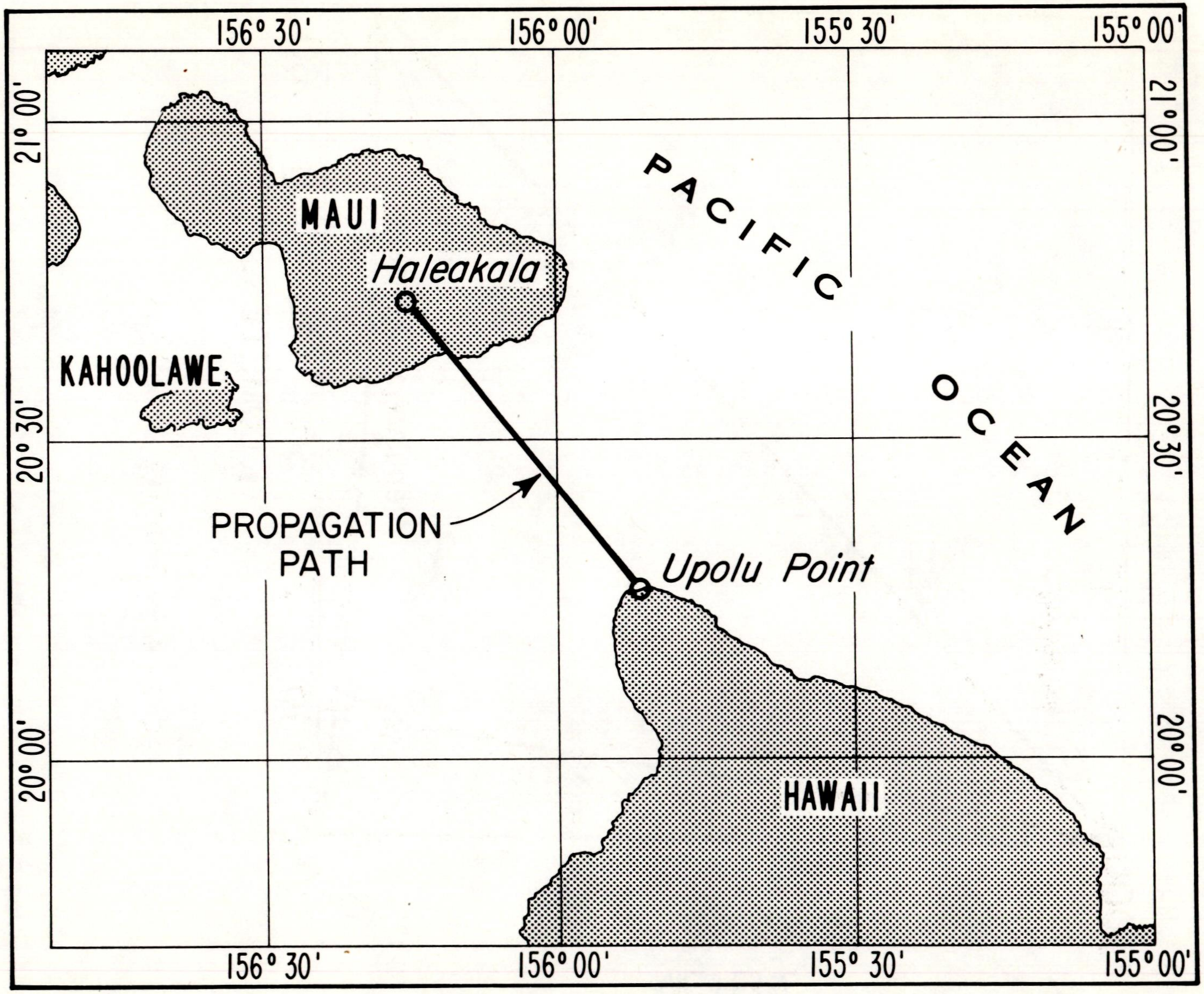

Figure 1. Map of path.

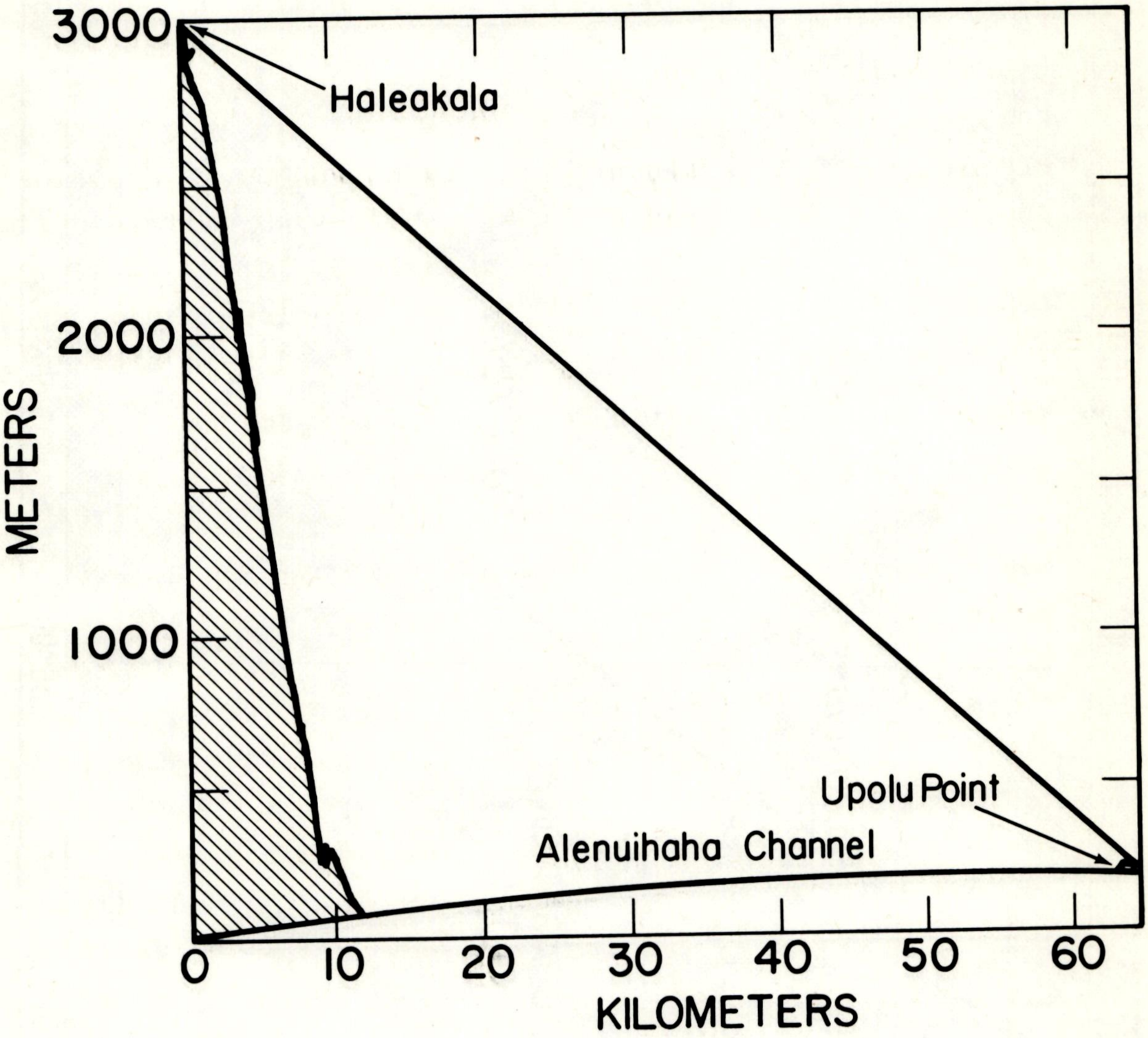

Figure 2. Profile of path.

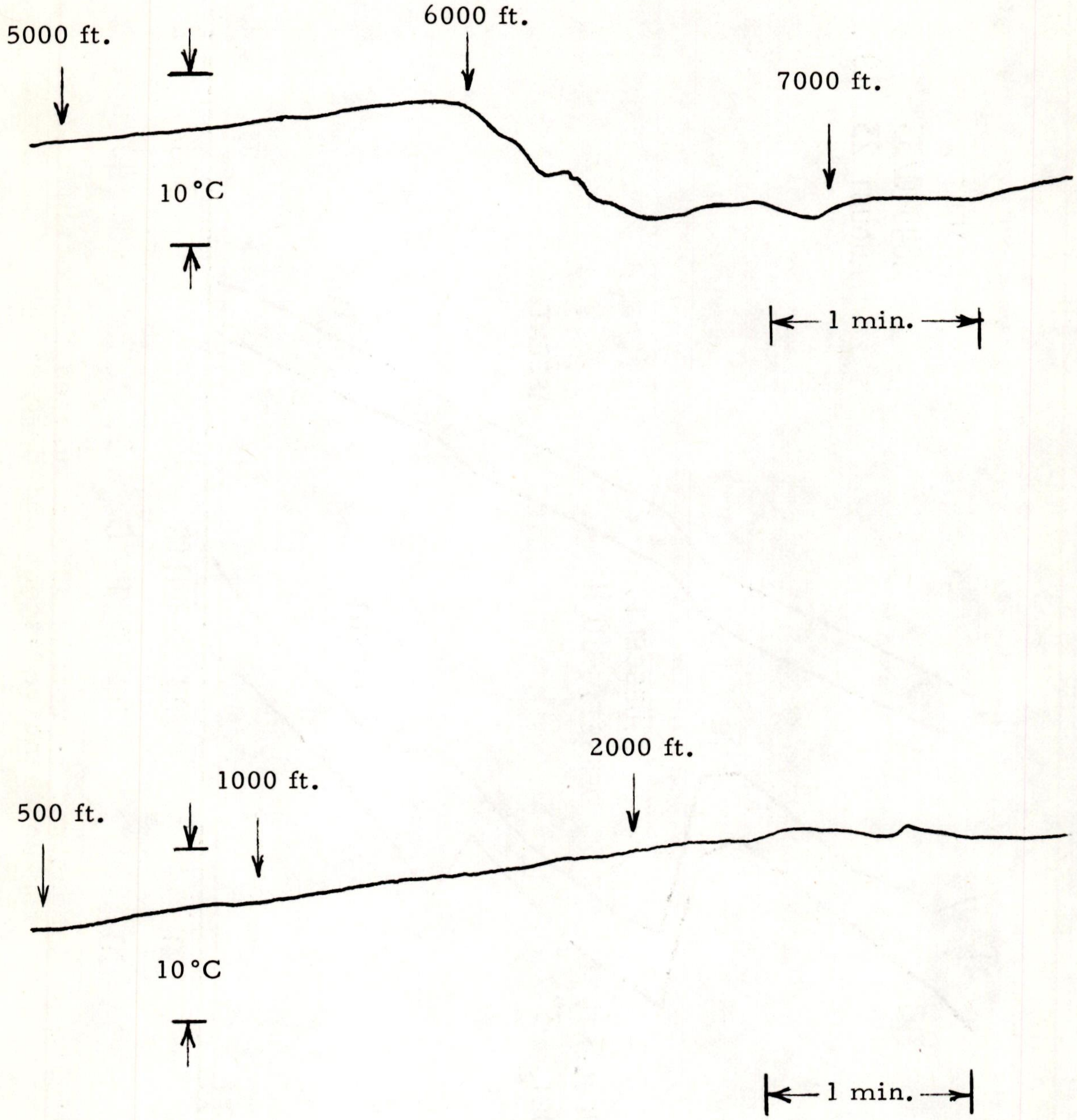

Figure 3. Sample temperature recordings.

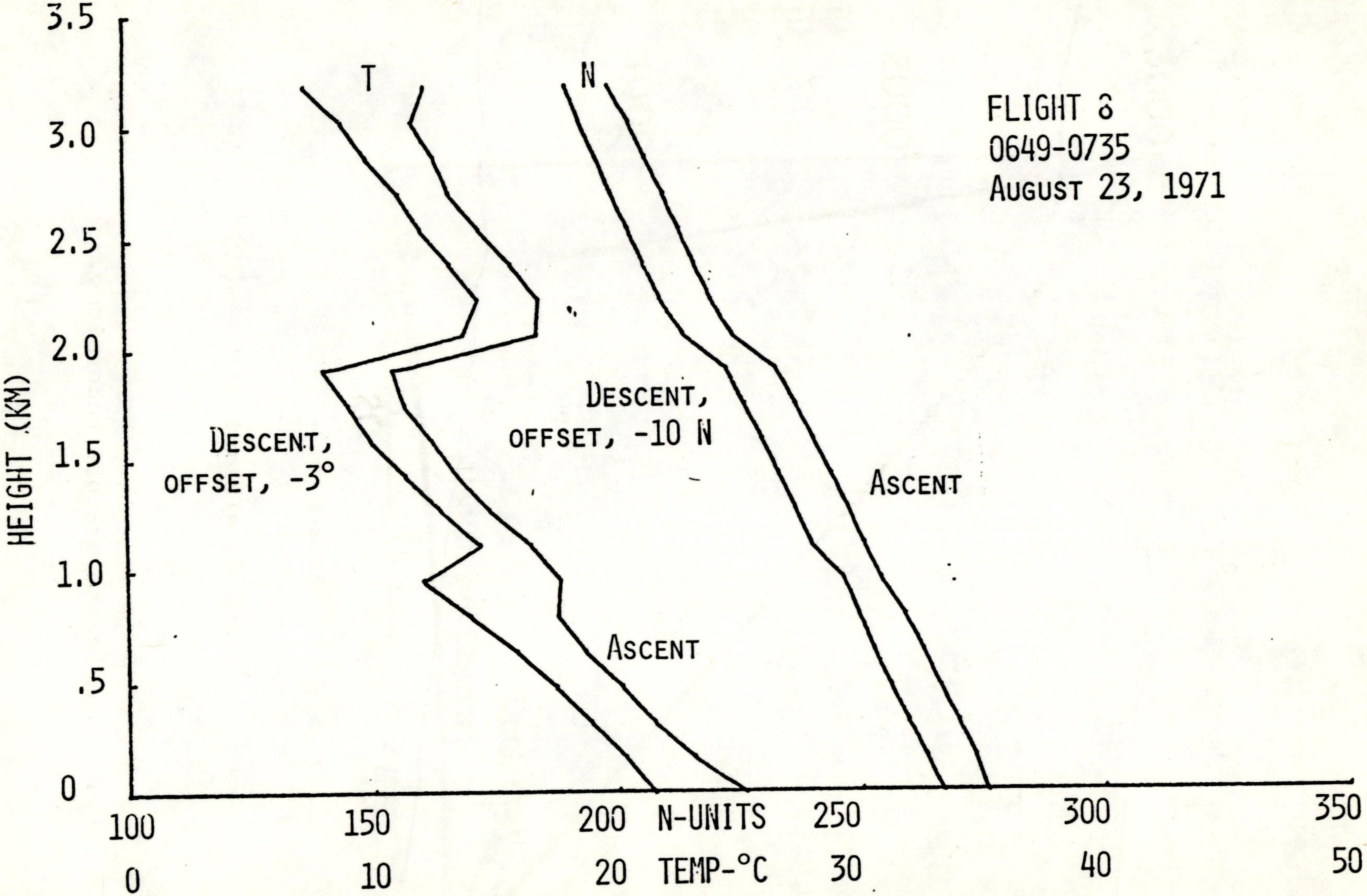

Figure 4. Measured T-profile and calculated N-profile for consecutive ascent and descent.

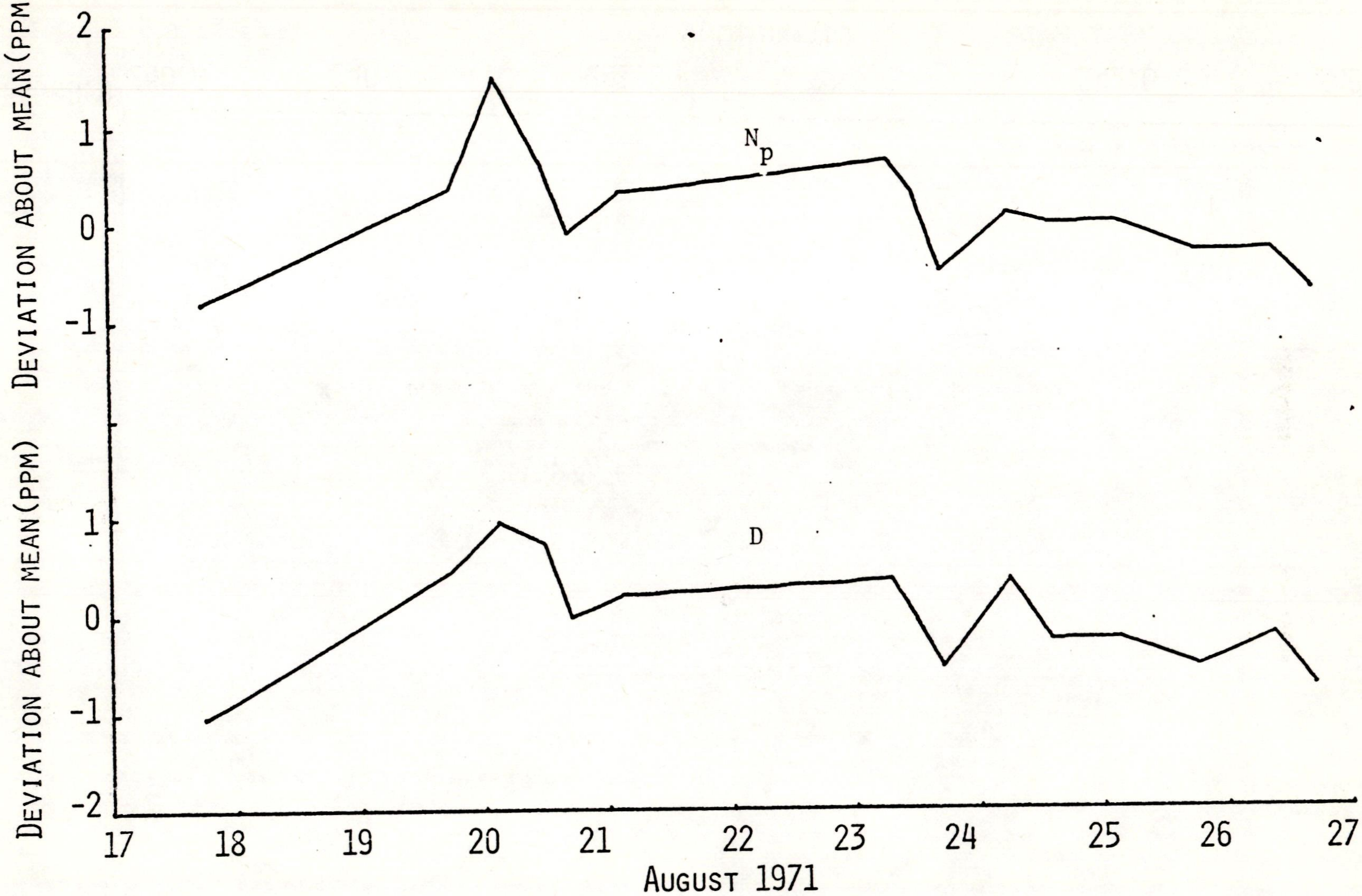

Figure 5. Time variations of average N along path calculated from ray-tracing and measured length, D.

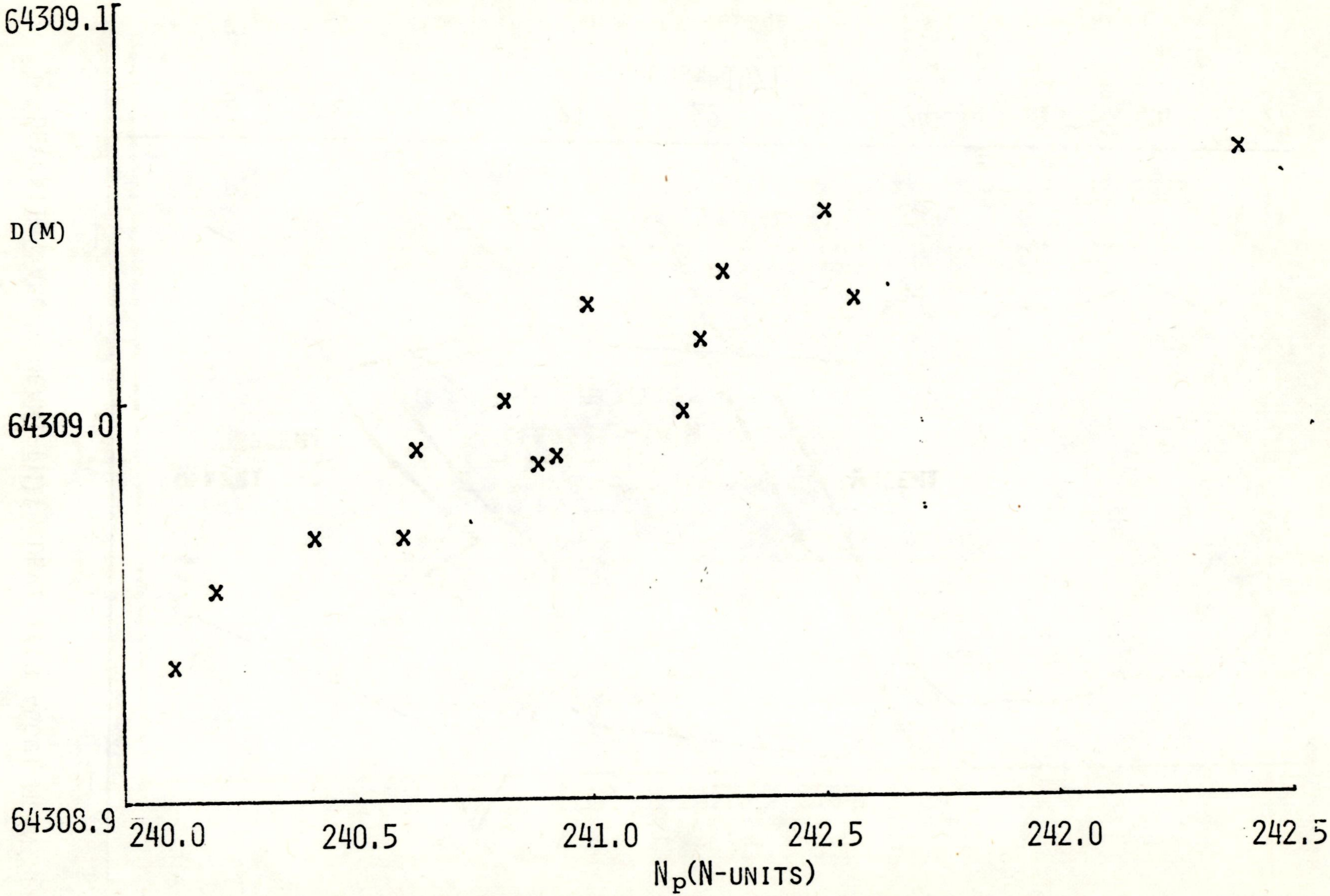

Figure 6. Cross-plot of measured length, D, and average N along path calculated from ray-tracing.

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15. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

A 64 km path between the islands of Hawaii and Maui was measured using a Geodimeter with refractive index correction based on airborne measurements of temperature along the line. The standard deviation of the slant range is estimated as about 0.9 ppm (0.06 m).

16. Key words *(Alphabetical order, separated by semicolons)*

Calibrated range; refraction correction.

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